DYNAMIC BEHAVIOR OF COOLING TOWERS UNDER GROUND ACCELERATION AND REAL TIME MOTION VISUALIZATION

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Abstract. This paper summarizes the background of linear and non-linear dynamic earthquake analysis methods for shells of revolution. Special attention is paid to the dynamic behavior of thin cooling tower shells (special type of shells of revolution) subjected to earthquake ground motion. Here, different types of cooling tower structures with different dynamic behavior – such as natural draught cooling towers or fan supported cooling towers – are considered in application examples. Different dynamic behavior and non-linear effects mainly due to the non-linear behavior of reinforced concrete are demonstrated with special emphasis on the practical design. The presented algorithms are implemented in computer code FEMAS – ROSHE which has been successfully applied to the analysis and design of a large number of cooling towers worldwide. Another very important aspect in dynamic analysis of engineering structures is the visualization and post processing of background data and results such as forces, stresses, strains. Naturally, in dynamic earthquake analysis of cooling towers a large number of time steps (= 2000 - 4000) has to be analyzed for a given ground motion time history. Theory and application of powerful MATLAB graphic toolbox are presented for 3D visualization of results and creation of real time motion movies which can be advantageously used for better understanding of structural response and mechanical behavior.
1 INTRODUCTION

In power plant engineering shells of revolution are frequently found as structural systems. Special representatives are pressure vessels, containments, tanks, fly ash silos and others. Among those representatives, cooling towers belong to the most fascinating and demanding structures due to their height, thin wall thickness and negative curved surface.

The geometry of shells of revolution as a special group of arbitrary double curved spatial shell structures is described by a mathematical relation between the height $Z$ and the radius $R(Z)$ of the shell. From this relation all geometric properties such as metric, curvature, local base vectors and their derivatives can be obtained. For the numerical calculation of shells of revolution with axis symmetric geometry, ring elements can be deployed advantageously within a finite element concept, both for linear and non-linear static and dynamic structural response (e.g. consideration of realistic properties of reinforced concrete such as cracking and crushing).

Research on ring elements originally comes from aeronautical and space structure engineering and dates back to the early works in the 1960s, e.g. [6]. First attempts for consideration of non-linearity of steel material in ring elements within a pseudo-load formulation have been undertaken in the beginning of the 1980's [9] for elasto-plastic steel material. In this contribution, the non-linear formulation of an existing linear shell ring element [2] with consideration of realistic material laws for reinforced concrete is briefly discussed with aspects of non-linear dynamics. With this theoretical background, the earthquake behavior and design of different types of modern cooling towers (both natural draught and fan supported cooling towers) against earthquake acceleration is studied.

The visualization of computer results becomes more and more important since application of computers in the analysis of structures has increased exponentially in the last decades. Within a non-linear dynamic simulation, a huge number of time steps is analyzed and has to be checked. Therefore, a real time motion simulation utility with stress plotting option is strongly required to evaluate and interpret results of transient dynamic analysis. In this contribution, very simple but powerful routines are presented how to use MATLAB plotting capabilities to generate real time motion animations and to visualize results from numerical output data.

2 GENERAL ASPECTS OF CONSTITUTIVE MODELLING OF REINFORCED CONCRETE

In Figure 1, a cut-out of a shell of revolution with its body forces and stress resultants is depicted. By numerical section integration through the depth, membrane forces $n_{\alpha\beta}$ and moments $m_{\alpha\beta}$ are computed using Equation (1) and (2) for arbitrary constitutive behavior. Transversal shear forces are not of interest in this formulation since the Kirchhoff-Love restriction applies for this special type of problems considered here. There is no need for inclusion of shear deformations for this type of problems.

Three different types of material laws are used for the modeling of a reinforced concrete section: a) uniaxial material laws in direction of the reinforcing steel bars, b) a biaxial non-linear isotropic plane stress material law for uncracked concrete and c) orthotropic material laws for tension stiffening after cracking.

$$n_{\alpha\beta} = \int_{-h/2}^{+h/2} \mu(\theta^3) \cdot [\delta_{\alpha}^\beta - \theta^3 \cdot b_{\alpha}^\beta] \cdot \sigma^{\alpha\lambda}(\theta^3) \, d\theta^3$$  

(1)
\[ m^{\alpha\beta} = \int_{-h/2}^{h/2} \mu(\theta^3) \cdot \Phi^3 \cdot \sigma^{\alpha\beta}(\theta^3) \, d\theta^3 \]  

(2)

3  FE MODELING OF SHELLS OF REVOLUTION WITH RING ELEMENTS

3.1  Geometry and displacement interpolation

For the numerical analysis of shells of revolution ring elements can be used advantageously. In Figure 2, the geometry of a shell ring element is depicted as implemented in finite element code ROSHE [8]. The middle surface is described by a relation between height Z and radius R(Z). With exclusion of non-symmetric terms, the displacement vector is expressed using a Fourier series with order \( n_{\text{max}} \) according to Equation (3).

For the unknown Fourier coefficients of displacements \( u_i \), a finite element approximation is introduced. Cubical polynomials in meridian direction \( \theta^2 \) are used for each component of the displacement vector. Hence, the proposed ring element possesses 12 DOFs per Fourier term as depicted in Figure 2.
\[ \mathbf{u}(\theta^1, \theta^2) = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \sum_{n=0}^{n_{\text{max}}} \begin{bmatrix} u_1(\theta^2) \cdot \sin(n\theta^1) \\ u_2(\theta^2) \cdot \cos(n\theta^1) \\ u_3(\theta^2) \cdot \cos(n\theta^1) \end{bmatrix} \] (3)

3.2 Principle of virtual work for dynamic problems

The principle of virtual work implies the equilibrium equations in weak formulation and is given in Equation (4) for a shell with some simplifications. For dynamic problems, additional inertia and damping forces have to be considered as described by the last two expressions.

\[
\begin{align*}
\int_A \int_\mathcal{C} \left( p^\alpha \delta u_\alpha + p^3 \delta u_3 \right) dA \\
+ \int_A \int_\mathcal{C} \left( n^\alpha \delta u_\alpha + n^3 \delta u_3 \right) dC \\
- \int_A \int_\mathcal{C} \left( \tilde{n}^{\alpha\beta} \delta \alpha_{\alpha\beta} + m^{\alpha\beta} \delta \beta_{\alpha\beta} \right) dA \\
- \int_A \int_\mathcal{C} \left[ c_1 a^{\alpha\beta} \delta u_\beta + c_n \delta u_3 \right] dA \\
- \int_A \int_\mathcal{C} [c_1 a^{\alpha\beta} \delta u_\beta + c_n \delta u_3] dA = 0
\end{align*}
\] (4)

For a system with discrete DOFs, this principle yields the well known equations of motion according to Equation (5) which can be identified as dynamic equilibrium where \( \mathbf{P}_e = \) vector of applied forces; \( \mathbf{P}_i = \) vector of resisting forces; \( \mathbf{P}_m = \) vector of inertia forces; \( \mathbf{P}_{ce} = \) vector of external damping forces.

\[ \mathbf{P}_e - \mathbf{P}_i - \mathbf{P}_m - \mathbf{P}_{ce} = 0 \] (5)

In general, Equation (5) cannot be solved directly due to non-linear behavior and therefore has to be linearized. A split-up of all mechanical variables into a current state and increments is required. After introducing the FE approximation into the principle of virtual work, the element vectors and matrices for the ring element are obtained. For linear structural response, all matrices are decoupled in Fourier terms due to orthogonality properties of harmonic functions. Coupling of harmonic displacement functions occurs in the tangent stiffness and internal damping matrix with increasing non-linearity.

4 SOLUTION OF EQUATIONS OF MOTION AND MOTION VISUALISATION

4.1 Solution of transient non-linear equations of motion

In non-linear dynamics, the system of equations of motion is expressed by Equation (5). However, this equation cannot be solved directly due to the dependence on the current state but has to be linearized. Incremental iterative solution strategies have to be applied to the linearized system of equations as given in Equation (6).

\[ \mathbf{K}_t \cdot \mathbf{\dot{v}} + \mathbf{C} \cdot \mathbf{\dot{v}} + \mathbf{M} \cdot \mathbf{\ddot{v}} = \mathbf{P}_e - \mathbf{P}_i - \mathbf{P}_{ce} - \mathbf{P}_m \] (6)
For non-linear structural behavior, direct integration schemes work almost the same as for linear behavior. The difference consists in the formulation in displacement, velocity and acceleration increments. From a given state of equilibrium of the last time step, the time is increased which results in a change of effective load due to the change in the external forces and due to the motion. The first solution of the new variables of motion for a new time step is calculated in the increment step (predictor step). After that, equilibrium is checked in the updated state. However, it is most likely not to be satisfied due to the linearization error. Iteration steps (or corrector steps) are applied until the convergence criterion is satisfied (out of balance forces must vanish). Actually, in every increment or corrector step the problem is reduced to the problem of a linear static analysis using an effective stiffness which consists of a combination of stiffness, mass and damping and an effective load vector.

The non-linear problem cannot be decoupled in Fourier terms. However, these coupling off-diagonal terms are of minor importance in the effective stiffness matrix due to contributions of the super-diagonal mass matrix and Rayleigh damping matrix and can be neglected in the solution process (modified Newton type algorithm). Hence, the use of direct integration schemes in combination with ring elements becomes very effective. The importance of mass contribution even increases quadratically with smaller time steps.

For horizontal earthquake acceleration, the equivalent vector of body forces is shown in Figure 3 for a shell of revolution. From these equivalent body forces the equivalent vector of external loads $P_e$ is derived using the first term of Equation (6).

\[
\begin{align*}
\mathbf{p}_{e}^{<1>} &= \rho \cdot \mathbf{h} \cdot \dot{\mathbf{v}}_{gr}(t) \cdot \sin(\theta') \\
\mathbf{p}_{e}^{<3>} &= -\rho \cdot \mathbf{h} \cdot \dot{\mathbf{v}}_{gr}(t) \cdot \cos(\theta')
\end{align*}
\]

Figure 3: Equivalent vector of body forces due to ground motion.

4.2 Algorithms for visualization of motion

As mentioned before the visualization of results and motion becomes very important for the understanding and interpretation of mechanical behavior of structures. Therefore numerical output data from finite element analysis has to be translated in moving pictures.
This task can be performed very simple and efficient using the powerful graphic capabilities of MATLAB. For the creation of real time movies, the algorithm is described in the following for the creation of eigenmode movies.

```matlab
nopica=input('
 Number of pictures per period: ');
for ip=1:nopica
    MF=MAGF*sin(2*PI/nopica*(ip-1));
    close all;
    axis ('equal','off');
    view(-180,kippw);
    PlotSytem;
    Mpip(ip)=getindexedframe(gcf);
end;
aviwrite('Mpic.avi',Mpic,25);
```

For the motion in a given time interval, a given eigenvector is scaled with \( \sin(\omega t) \). With a given number of pictures `nopica` the eigenvector is scaled with different magnification factors within a loop. Each time a new picture is generated using subroutine `PlotSystem`. These pictures are captured using command `getindexedframe` and stored in an array of pictures `Mpic` in each step. After creation and storing of all plots which describe the motion the array of pictures `Mpic` is saved into an avi-file using the command `aviwrite`. This avi file can be played with almost any computer media player.

In case of creation of real time motion movies, displacements in every time step and corresponding results (such as strains, stresses, forces) have to be stored. For every time step, a new picture has to be created. Colored pictures can be easily created using the `patch` command. This command requires the arrays of coordinates (x-, y-, z-coordinates) and corresponding results which shall be plotted for one closed surface (e.g. four node rectangular surface).

```matlab
h=patch(x_lok,y_lok,z_lok,result);
```

The creation of avi-movies again is simply done by capturing each picture in each time step and dumping this array of pictures into an avi-file by using the `aviwrite` command. Thus, the most effort has to be spent in the preparation and transformation of numerical output data. The actual plotting process benefits from powerful graphic commands as described before. The described graphic capabilities are used in the following for the analysis and visualization of application studies.

5 APPLICATION STUDY FOR NATURAL DRAUGHT COOLING TOWER

5.1 Natural draught cooling tower

The first application study is a modern natural draught cooling tower with total height of approximately 180 m and a wall thickness of 20 cm in most parts as depicted in Figure 4. The tower is supported by a system of 46 V-columns with diameter \( \varnothing 90 \) at its base. This system is taken as a reference system for earthquake analysis of a typical modern natural cooling tower concerning dimensions and wall thickness although no earthquake design requirements had to be fulfilled for this tower.

As described before ring elements with Fourier approximation in hoop direction are deployed for the structural modeling of shells of revolution. The upper ring beam is modeled by a ring beam element. The column support is modeled by a special macro truss column ele-
ment. This macro column element consists of single 3D beam elements which are arranged in space. They are coupled at their upper and lower node in nodal circles. The deformation of these nodal circles is again described by Fourier series.

![Figure 4: Geometry of natural draught cooling tower and FE-model.](image)

Although no earthquake analysis was required in the original design this system is analysed for the earthquake combination

$$ g + 3.5 \text{ m/s}^2 \cdot f(t) $$

where $f(t)$ is a unit scaled time history acceleration function.

A maximum acceleration of 3.5 m/s$^2$ as base acceleration is assumed. Different structural methods are used to analyze the tower for this given ground acceleration:

- a) Equivalent Static Acceleration (according to first relevant period of the system)
- b) Response Spectrum Method
- c) Linear dynamic transient analysis for different ground motion time histories $f(t)$
- d) Non-linear dynamic transient analysis for different ground motion time histories $f(t)$

For transient dynamic analysis, different ground motion histories are used: El Centro, Kern County, Loma Prieta, San Fernando and Tabas earthquake (details see [5]). The following results can be summarized for the system of this large natural draught cooling tower on V-columns.
The design of the shell is not governed by the earthquake combination in general. Here, minimum reinforcement requirements (formation of first cracks) in combination with ULS combination \( g + 1.60 \) \( w \) become decisive for the design of the shell. Only the lower part of the shell, which is directly influenced by the load introduction of column forces has to be considered separately for earthquake design situation.

However, column tensile forces are much higher in earthquake combination than in wind combination. For standard wind design combination \( g + 1.60 \) \( w \) without accounting for earthquake design minimum reinforcement section \( a_s = 50 \) \( \text{cm}^2 \) governs the design. This minimum reinforcement covers the design against wind forces in ULS.

With consideration of earthquake resistant design against ground acceleration 3.5 m/s\(^2\) the required reinforcement increases approximately by factor 3 using linear elastic analysis (equivalent static force, response spectrum method, linear dynamic transient analysis). Good agreement between different methods is obtained. The design forces result in very strong column reinforcement (approx. 2.5% of the total section) with corresponding problems (anchoring of reinforcement in shell, overlapping of reinforcement in columns, decrease of ductility).

It should be noted that non-linear effects such as uplift of ring foundation from soil, cracking of concrete of shell and cracking of columns due to tensile forces occurs. These non-linear dynamic effects result in the following main aspect: The column tensile forces are reducing significantly so that the column reinforcement for earthquake resistant design can be reduced simultaneously.

Design results for the V-columns \( \varnothing 90 \) concerning reinforcement ratios are summarized in Figure 5 schematically. Non-linear effects (mainly uplift and cracking of columns) provide a strong potential of stress reduction which results in lower reinforcement amounts. According to the conducted non-linear dynamic transient studies, a behavior factor of \( q \approx 1.50 \) is found for this system. Local dissipative effects (e.g. bond behaviour of column reinforcement anchoring in shell under cyclic loading), however, are not considered in these global studies which provide further energy dissipation mechanisms.

\[
\begin{align*}
\text{g+1.60 w} & \quad \gamma_s = 1.15 \\
16\varnothing20 &= 50 \text{ cm}^2 (0.8\%) \\
\text{Minimum } a_s \\
\text{g+3.5 m/s}^2 & \quad \gamma_s = 1.00 \\
50\varnothing20 &= 157 \text{ cm}^2 (2.5\%) \\
\text{LINEAR} \\
\text{g+3.5 m/s}^2 & \quad \gamma_s = 1.00 \\
29\varnothing20 &= 91 \text{ cm}^2 (1.4\%) \\
\text{Kern County} \\
\text{NON-LINEAR}
\end{align*}
\]

*Figure 5: Comparison of reinforcement quantities.*
5.2 Fan supported cooling tower

The second application study [4] covers a fan supported cooling tower on I-beam columns as depicted in Figure 6. In this case the hole panels between the I-beams have no static function, they are supplemented afterwards. All forces are carried by the I-beam support.

![Fan supported cooling tower](image)

This tower has been erected in an earthquake sensitive zone with high ground acceleration. The column design has been governed by ULS earthquake combination. The design free field acceleration has been determined with 3.50 m/s² with the corresponding response spectra according to Eurocode 8 as shown in Figure 7. A maximum elastic response acceleration of an elastic SDOF system is obtained with 8.50 m/s².

![Design spectra for fan supported cooling tower](image)
The first period of the system is $T = 0.57$ s. However, this period and corresponding eigenvector is not of interest for earthquake design since it describes a vibration of the shell with $n=4$ circumferential waves. The first governing eigenvector for earthquake horizontal ground motion with circumferential wave pattern $n=1$ can be found with period $T = 0.41$ s. These eigenvectors are depicted in Figure 8.

![Figure 8: Natural eigenmodes of fan supported cooling tower.](image)

The governing design combination for columns including an importance factor of $\gamma_I = 1.40$ thus becomes

$$g + \gamma_I \cdot e / q \quad \text{with} \quad \gamma_I = 1.40 \quad \text{and} \quad e = 8.5 \text{ m/s}^2 \quad \text{and} \quad q = 3.0$$

while the governing eigenperiod $T = 0.41$ s exactly matches the interval of periods with maximum dynamic amplification according to Figure 7. However, for this structure with I-beams (mainly bending) a larger behavior factor can be applied than for the system with V-truss columns (mainly pure tension and compression). For this special type of structure, a behavior factor of $q \approx 3.0$ can be justified with reference to design codes or by application of capacity design method since the ductility is higher due to the bending of I-columns for the transfer of horizontal loads than for the system with V-columns. This results in an equivalent static acceleration $a = 1.40 \times 8.50 / 3.00 = 3.97 \text{ m/s}^2 \approx 4 \text{ m/s}^2$.

As mentioned before, the shell design is not governed by earthquake combination (only load introduction of column forces and moments at base support). However, huge horizontal forces due to the acceleration of the total shell mass have to be transferred from the shell into the foundation by the meridional I-beams. The mass of the shell can be regarded as a rigid body which is rocking on the elastic column support as can be obtained from the shape of the corresponding eigenmode. Thus, the analysis in this case is quite simple: No superposition of different modes or transient dynamic analysis has to be carried out for the design. The column axial and transversal forces plus corresponding bending moments can be simply obtained by application of the equivalent static acceleration for the whole structure (in this case $\approx 4 \text{ m/s}^2$).

This procedure where account of non-linear effects are already included by application of behavior factor $q = 3$ leads to a column design which is just close to the limit of technical feasibility (e.g. splices of reinforcement, anchoring of reinforcement in shell). The corresponding design result is shown for the upper column section at the transition to the shell in Figure 9.
Here, a reinforcement quantity of approximately 4% of the total section has to be installed to equilibrate the forces and bending moments generated by earthquake, although this is just a tower with very small dimensions in height.

Figure 9: Reinforcement design results for fan supported tower.

6 SUMMARY AND CONCLUSIONS

In this paper, a non-linear formulation of ring elements has been presented for non-linear dynamic earthquake analysis of shells of revolution. The basic aspects of the derivation of element matrices have been discussed. This ring element concept can be easily applied to other structural members (e.g. ring spring element, macro column element, and ring beam element). Further, methods for solving the equations of motion of a shell of revolution under horizontal earthquake acceleration have been shortly discussed. With these methods, shells of revolution with axissymmetric geometry can be analyzed in a very simple but highly effective way using linear and non-linear static and dynamic analysis and design concepts.

The plotting and visualization of results becomes very important especially for transient dynamic analysis. A very simple way of using MATLAB graphic library tools is presented for this purpose. With these simple routines, large amount of result background data from finite element analysis can be postprocessed and animated very efficiently.

With the presented tools and numerical algorithms, two application studies of different types of cooling towers are analyzed for horizontal earthquake motion. Due to their different structural composition the earthquake load bearing behavior and structural response is completely different for these two systems. The results of earthquake design are presented in terms of column reinforcement detailing. Non-linear effects have been demonstrated briefly. Their consideration in the structural earthquake design is absolutely necessary to come to a technically feasible design result which can be executed in engineering practice.

In the conference presentation, the design results and dynamic behavior of the analyzed towers will be supplemented and supported by visualized animations.
REFERENCES


[10] MATLAB User Reference